A TOA-based Location Algorithm Reducing the Errors Due to Non-Line-of-Sight (NLOS) Propagation

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Abstract—An effective location algorithm, which considers Non-Line-of-Sight (NLOS) propagation, is presented. By using a new variable to replace the square term and adding the loose variable, the problem becomes a mathematical programming problem, and then the NLOS propagation’s effect can be evaluated. This method is simple and does not add much computation time. Compared with other methods, it has high accuracy.

I. INTRODUCTION

In land cellular wireless location systems, one of the key parts is the location algorithm, which is based on TOA or TDOA measurements[1–3]. However, in the multipath propagation environment, the data of TOA are often obtained from one of the non-directive paths. So, the accuracy of the location algorithm is degraded severely due to the NLOS (non-line-of-sight) propagation in multipath environment.

A two-step maximum-likelihood (ML) TDOA-based location algorithm proposed by Chan and Ho has high accuracy when the NLOS propagation interference is not very serious[4]. Also, other methods[5–8] didn’t consider the inferences of NLOS propagation on the location estimation. A NLOS location algorithm is presented in [9], but it depends on a probability density function (PDF) which is given by a model based on plenty of practical measurements. So this NLOS algorithm is difficult to be applied because the PDF should be changed in different environments.

Thus, in the situation that the receivers are distributed arbitrarily, we present a TOA-based location algorithm in which the errors introduced by NLOS propagation can be reduced. According to the simulation and comparison, this method shows higher accuracy and does not need significant computation.

\[ r_i^2 \leq (x_i - x)^2 + (y_i - y)^2 = K_i - 2x_i x - 2y_i y + x^2 + y^2, \]
\[ i = 1, 2, \cdots, M. \]

Inserting loose variables \( v_i \) and let variable \( R = x^2 + y^2 \), then we have a set of nonlinear equations again
\[ r_i^2 - K_i = -2x_i x - 2y_i y + R + v_i, \]
\[ 0 \leq v_i \leq r_i^2, \quad i = 1, 2, \cdots, M. \]

We assume that only M-3 added loose variables are the unknown variables and assume the remaining 3 loose variables as known parameters, then...
\[ r_i^2 - K_i = -2x_i x - 2y_i y + R + v_i, \quad i = 1, 2, \cdots, M - 3, \]
\[ r_i^2 - K_i - v_i = -2x_i x - 2y_i y + R, \quad i = M - 2, M - 1, M. \]
\[ 0 \leq v_i \leq r_i. \]

Let \( Z_a = [x, y, R, v_1, \cdots, v_{M-3}]^T \), and error vector
\[ \psi = h - G_a Z_a, \]
where
\[ h = \begin{bmatrix}
  r_1^2 - K_1 \\
  \vdots \\
  r_{M-2}^2 - K_{M-2} - v_{M-2} \\
  r_{M-1}^2 - K_{M-1} - v_{M-1} \\
  r_M^2 - K_M - v_M
\end{bmatrix}, \]
\[ G_a = \begin{bmatrix}
  -2x_1 & -2y_1 & 1 & \cdots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  -2x_{M-3} & -2y_{M-3} & 1 & \cdots & 1 \\
  -2x_{M-2} & -2y_{M-2} & 0 & \cdots & 0 \\
  -2x_{M-1} & -2y_{M-1} & 0 & \cdots & 0 \\
  -2x_M & -2y_M & 0 & \cdots & 0
\end{bmatrix}. \]

Then we still use ML method to estimate and have
\[ Z_a = \arg \min \{ \| h - G_a Z_a \| \} = (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1} h, \]
where
\[ \Psi = E[\psi \psi^T] = 4\sigma^2 B B^T, \]
and \( B = \text{diag}(\sigma_1^2, \cdots, \sigma_M^2) \), and \( Q \) is the covariance matrix of measured noise.

Obviously, \( Z_a \) is not a group of constants but a function of \( [v_{M-2}, v_{M-1}, v_M]^T \). Instead, it can be expressed as
\[ Z_a = Z_{ac} + C \begin{bmatrix} v_{M-2} \\ v_{M-1} \\ v_M \end{bmatrix}, \]
(1)

where
\[ Z_{ac} = (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1} \begin{bmatrix}
  r_1^2 - K_1 \\
  \vdots \\
  r_{M-2}^2 - K_{M-2} \\
  r_{M-1}^2 - K_{M-1} \\
  r_M^2 - K_M
\end{bmatrix}, \]
\[ C = (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1}. \]

Because \( v_1, \cdots, v_{M-3} \) become functions of \( \begin{bmatrix} v_{M-2}, v_{M-1}, v_M \end{bmatrix}^T \), the solution space of the demand location position can be rewritten as
\[ \begin{bmatrix}
  -C' \\
  C'
\end{bmatrix} \begin{bmatrix} v_{M-2} \\ v_{M-1} \\ v_M \end{bmatrix} \leq \begin{bmatrix} Z_{ac} \\ v_r - Z_{ac} \end{bmatrix}, \]
\[ 0 \leq v_{M-2} \leq r_{M-2}^2, 0 \leq v_{M-1} \leq r_{M-1}^2, 0 \leq v_M \leq r_M^2 \]

where \( Z_{ac} \) is composed of the fourth to the \( M \)th elements of \( Z_{ac} \), \( C' \) is composed of the fourth row to the \( M \)th row elements of \( C \).

In most situations, the scope of the solution space is very large. So here, in the solution space, we search the closest position to the solution of the algorithm without NLOS consideration to get a more accurate solution. Assuming the position solved by the algorithm without NLOS consideration is \( (x', y') \), we can set up a cost function \( (x - x')^2 + (y - y')^2 \), and the problem is transformed to a mathematical programming problem:
\[
\begin{aligned}
\min & \quad (x - x')^2 + (y - y')^2, \\
\text{s.t.} & \quad \begin{bmatrix}
  -C' \\
  C'
\end{bmatrix} \begin{bmatrix} v_{M-2} \\ v_{M-1} \\ v_M \end{bmatrix} \leq \begin{bmatrix} Z_{ac} \\ v_r - Z_{ac} \end{bmatrix}, \\
0 \leq v_{M-2} \leq r_{M-2}^2, 0 \leq v_{M-1} \leq r_{M-1}^2, 0 \leq v_M \leq r_M^2
\end{aligned}
\]

where
\[ \begin{bmatrix}
  x \\
  y
\end{bmatrix} = C' \begin{bmatrix} v_{M-2} \\ v_{M-1} \\ v_M \end{bmatrix} + Z_{ac}. \]

where \( Z_{ac} \) is composed of the first to the second elements of \( Z_{ac} \), \( C' \) is composed of the first to the second row.
elements of $\mathcal{C}$.

When the satisfying solution of (2) is found, substitute it into (1) to obtain $Z_a$ and the covariance matrix of $Z_a$

$$\text{cov}(Z_a) = E[\Delta Z_a \Delta Z_a^T] = (G_a \Psi^{-1} G_a)^{-1}.$$ 

Because there is relationship among $x, y, R$, we must impose this relation to get further estimation. Let the estimation errors of $x, y, R$ be $e_1, e_2, e_3$, then the elements in vector $Z_a$ can be expressed as

$$[Z_a]_1 = x^0 + e_1, [Z_a]_2 = y^0 + e_2, [Z_a]_3 = R^0 + e_3,$$

and let another error vector $\Psi^* = h^* - G_a'' Z_p$, where

$$h^* = \begin{bmatrix} [Z_a]_1^2 \\ [Z_a]_2^2 \\ [Z_a]_3^2 \end{bmatrix}, \quad G_a'' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad Z_p = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix},$$

then

$$\Psi^* = \begin{bmatrix} 2x^0 e_1 + e_1^2 + 2e_1 \\ 2y^0 e_2 + e_2^2 + 2e_2 \\ 2x^0 e_1 + e_1^2 + 2e_1 \end{bmatrix}, \quad \Psi^* = e_3,$$

$$\Psi^* = 2y^0 e_2 + e_2^2 + 2e_2.$$

Hence the covariance matrix of $\Psi^*$ is given by

$$\Psi^* = E[\Psi^* \Psi^{*T}] = 4B^*O^*B^*,$$

where $O = [\text{cov}(Z_a)]_{13,13}$.

Similarly, the ML estimate of $Z_p$ is

$$Z_p = (G_a'' \Psi^{-1} G_a'')^{-1} (G_a'' \Psi^{-1} h^*),$$

$$= (G_a'' B^{-1} O^{-1} B^{-1} G_a'')^{-1} (G_a'' B^{-1} O^{-1} B^{-1} h^*).$$

So the final position estimate $Z = [x \ y]^T$ is

$$Z = \sqrt{Z_p}, \quad \text{or} \quad Z = -\sqrt{Z_p}.$$ 

Here the sign of $x$ coincides with the sign of the first element $[Z_a]_1$ of $Z_a$ calculated by (1), and the sign of $y$ coincides with the sign of the second element $[Z_a]_2$ of $Z_a$.

III. SIMULATION RESULTS AND COMPARISON

In estimation, assume that there are 10 receivers whose positions are $(x_0^0=0, y_0^0=0), (x_0^1=5, y_0^1=8), (x_0^2=4, y_0^2=6), (x_0^3=-2, y_0^3=4), (x_0^4=7, y_0^4=3), (x_0^5=-7, y_0^5=-5), (x_0^6=4, y_0^6=-2), (x_0^7=3, y_0^7=-3), (x_0^8=1, y_0^8=8)$, and the source move randomly in the square space $(-5,-5) \leq (x_0, y_0) \leq (5,5)$: $x_0 = 10* \text{rand} - 5, \quad y_0 = 10* \text{rand} - 5$.

Let

$$r_i = c(d_i + \Delta d_i) = cd_i + c\Delta d_i = r_i^0 + cn_i + N \cdot \text{rand}(\cdot),$$

where $\text{rand}(\cdot)$ is a random number from 0 to 1, $N$ is the possible maximum error induced by NLOS, and assume $cn_i$ is a Gaussian random noise which is zero mean with same variance $\sigma^2$.

It can be proved that the TOA-based location algorithm without NLOS consideration is optimal estimator and the estimation error can theoretically reach Cramer-Rao lower bound when there is no NLOS propagation[4]. So we can evaluate the performance of the NLOS algorithm through comparing the mean square errors (MSE) and the standard deviation (STD) of the LOS algorithm and the proposed NLOS algorithm. The MSE’s and STD’s of the NLOS algorithm and non-NLOS algorithm are compared in Table I, where their MSE’s and STD’s are changed with the number of receivers, $M$, used in localization. In Table I, taking $\sigma^2 = 0.01, N=3$, where $N$ is the possible maximum error induced by NLOS, the MSE $= E[(x-x)^2 + (y-y)^2]$ are obtained from the average of 10000 independent runs, STD is the square root of MSE. Shown as Table I, the MSE’s and STD’s increase as the number of the receivers $M$ decreases. The MSE’s and STD’s of the NLOS algorithm and LOS algorithm and their relationship with the power of Gaussian random noise, $\sigma^2$, are shown in Table II, where $N=3, M=10$, and similarly the MSE’s are obtained form the average of 10000 independent runs. Shown as Table II, the MSE’s and STD’s increase as the power of noise increases. And The MSE’s and STD’s of the NLOS algorithm and LOS algorithm and their relationship with the NLOS interference, $N$, are shown in Table III, where $M=10, \sigma^2 = 0.01$, the MSE’s are also obtained form the average of 10000 independent runs. Shown as Table III, the MSE’s and STD’s increase as the NLOS interference increases. Seen from all the tables, it’s obvious the MSE’s and STD’s of NLOS algorithm are significantly improved compared with those of non-NLOS algorithm. In the tables, A stands for
TOA-based LOS algorithm, B stands for TOA-based NLOS algorithm, M is the number of receivers, N is the possible maximum error induced by NLOS, $\sigma^2$ is the variance of Gaussian noise in the receivers.

**TABLE I**

<table>
<thead>
<tr>
<th>IN THE SITUATION $N=3$, $\sigma^2 = 0.001$</th>
<th>$\sigma^2$</th>
<th>$A$'s MSE</th>
<th>$B$'s MSE</th>
<th>$A$'s STD</th>
<th>$B$'s STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M=10$</td>
<td>0.0005</td>
<td>7.1401</td>
<td>1.2331</td>
<td>2.6721</td>
<td>1.1104</td>
</tr>
<tr>
<td>$M=9$</td>
<td>0.005</td>
<td>7.7895</td>
<td>1.3717</td>
<td>2.7910</td>
<td>1.1712</td>
</tr>
<tr>
<td>$M=8$</td>
<td>0.01</td>
<td>8.3575</td>
<td>1.6940</td>
<td>2.8910</td>
<td>1.3016</td>
</tr>
<tr>
<td>$M=7$</td>
<td>0.05</td>
<td>8.9837</td>
<td>1.9727</td>
<td>2.9973</td>
<td>1.4886</td>
</tr>
<tr>
<td>$M=6$</td>
<td>0.1</td>
<td>8.0953</td>
<td>2.2101</td>
<td>2.8452</td>
<td>1.4866</td>
</tr>
<tr>
<td>$M=5$</td>
<td>0.5</td>
<td>9.5543</td>
<td>3.0259</td>
<td>3.0910</td>
<td>1.7395</td>
</tr>
</tbody>
</table>

**TABLE II**

<table>
<thead>
<tr>
<th>IN THE SITUATION $N=3$, $M=10$</th>
<th>$\sigma^2 = 0.001$</th>
<th>$A$'s MSE</th>
<th>$B$'s MSE</th>
<th>$A$'s STD</th>
<th>$B$'s STD</th>
</tr>
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<td>7.1401</td>
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<td>1.7395</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>IN THE SITUATION $M=10$, $\sigma^2 = 0.01$</th>
<th>$A$'s MSE</th>
<th>$B$'s MSE</th>
<th>$A$'s STD</th>
<th>$B$'s STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N=0$</td>
<td>0.0958</td>
<td>0.0589</td>
<td>0.3094</td>
<td>0.2327</td>
</tr>
<tr>
<td>$N=1$</td>
<td>1.1618</td>
<td>0.2203</td>
<td>1.0779</td>
<td>0.4694</td>
</tr>
<tr>
<td>$N=2$</td>
<td>3.7569</td>
<td>0.6769</td>
<td>1.9383</td>
<td>0.8227</td>
</tr>
<tr>
<td>$N=3$</td>
<td>7.4023</td>
<td>1.3060</td>
<td>2.7207</td>
<td>1.1428</td>
</tr>
<tr>
<td>$N=4$</td>
<td>11.9319</td>
<td>2.1184</td>
<td>3.4543</td>
<td>1.4555</td>
</tr>
<tr>
<td>$N=5$</td>
<td>17.2962</td>
<td>3.2892</td>
<td>4.1589</td>
<td>1.8136</td>
</tr>
</tbody>
</table>

**IV. CONCLUSION**

In the multipath environment, the inference of NLOS must be considered in location estimation. The TOA-based NLOS location algorithm presented here can reduce the interference of NLOS propagation and reach high accuracy.

**REFERENCE**


